Investigating the transfer of mathematical knowledge using physicsless physics questions

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Research on the learning and teaching of thermal physics

Collaborators

Donald Mountcastle (U. Maine)

Warren Christensen (North Dakota St. U.)

Michael Loverude (California St. U. – Fullerton)

David Meltzer (Arizona St. U. Polytechnic)

DUE-0817282
PHY-0406764

DUE-0817335
PHY-0406724
PHY-0604703

- **Research on (mostly advanced) student understanding**
  - of thermal physics concepts
  - of the underlying mathematics and its application
- **Development of *student-centered* curricular materials**
  - to address specific difficulties found in research and literature
  - to provide students opportunity to wrestle with and integrate complex concepts

Research issue: Mathematics in physics

To what extent is student understanding of physics content affected by an understanding of the underlying ("prerequisite") mathematics?

- Making sense of mathematics and how it is used in physical situations

Physics ⇄ Math

Context for research

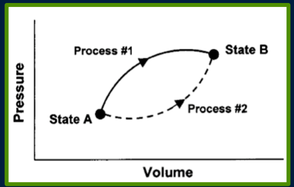
- Advanced undergraduate thermal physics courses;
  3 hours/week for semester
  - *Physical Thermodynamics (Carter)*
  - *Statistical Mechanics (Baierlein)*
  - Mathematics preparation

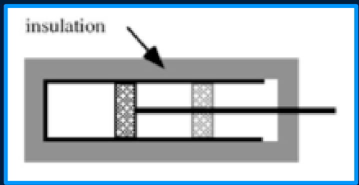
    All students had completed at least 4 mathematics courses, including *Calculus III* – multivariable differential calculus

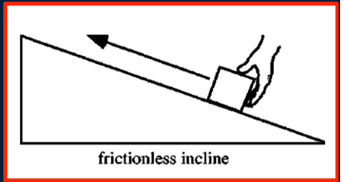
Quick review of student difficulties in physics: First Law of Thermodynamics

$$\Delta U = Q - W$$
$$(\Delta E = Q + W)$$

- connect ΔT to Q [1]
- don't discriminate between *T, Q, W, U* [1, 2]
- confusion among state functions, process variables [1,2] ("*change in* heat/work")
  - work independent of path / only depends on end states
  - work in cyclic process to be 0
- Over-reliance on the state function concept
  - Explicit assertion of path independence of work
  - Overgeneralization from conservative forces
- Many of the difficulties stem from absent or misguided attempts to connect thermodynamic work to mechanical work [1]



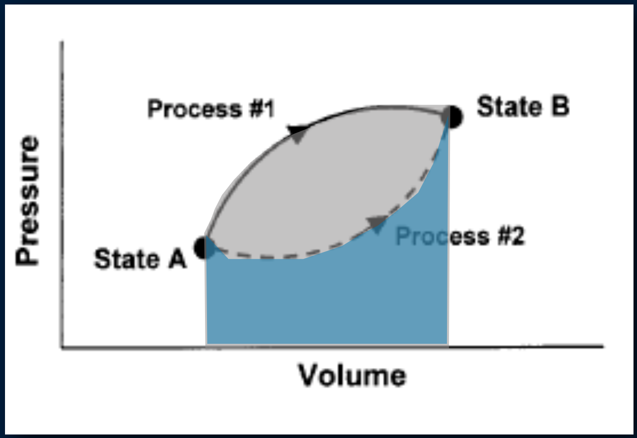




1. Loverude, Kautz, Heron, AJP 2002
2. Meltzer, *AJP* 2004

P-V diagrams (and the First Law of Thermodynamics)

Question from Meltzer, *Am. J. Phys.* 2004



"Is *W* for Process #1 *greater than, less than,* or *equal to* that for Process #2? Explain."

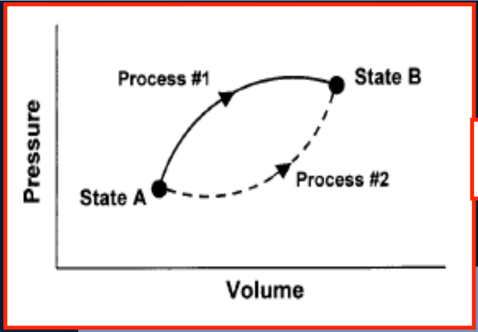
$$Work \equiv \int P dV$$

$$\Downarrow$$

<u>area under the curve</u>

for each process.

The physics question: Results from multiple disciplines



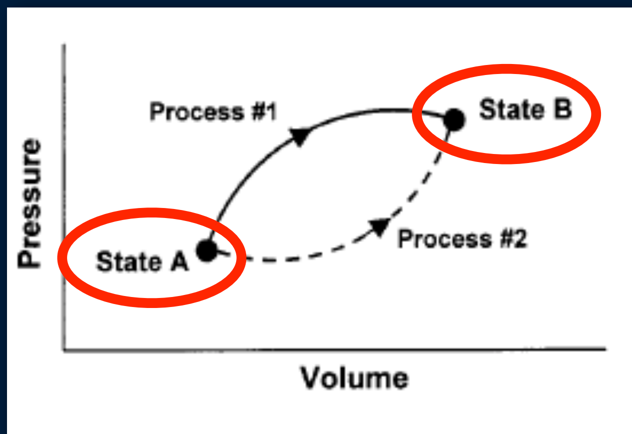
$W_1 \ >, <, = W_2$

Population	*Correct (ish) (+/– sign issues)*
Physical Thermodynamics Pre-instruction (N_{class}=2)	**5 / 15**
Physical Thermodynamics Post-instruction (N_{class}=3)	**14 / 20**
Physical Chemistry	2 / 8 *
Chemical Engineering Thermodynamics	34 / 41
AP Chemistry Teachers	2 / 9

*consistent with M.H. Towns and E.R. Grant, *J. Res. Sci. Teach.* **34**, 819-835 (1997)

P-V diagrams (and the First Law of Thermodynamics)

Question from Meltzer, *Am. J. Phys.* 2004



"Is W for Process #1 *greater than, less than,* or *equal to* that for Process #2? Explain."

$$Work \equiv \int PdV$$

$$\Downarrow$$

<u>area under the curve</u>

for each process.

***Common incorrect response: Works* equal**

(Intro level: Loverude et al., 2002; 25%-30% (Meltzer, 2004);
Upper level: ~40% (Pollock et al., 2007))

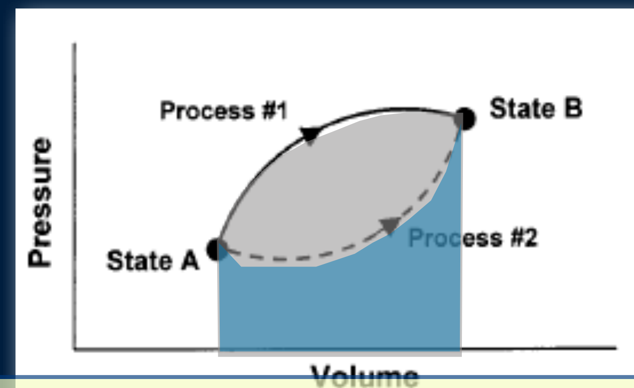
Common reasoning: Beginning and ending states are the same, so works are the same.

Interpretation: Students responding <u>as</u> <u>if</u> they are treating Work as a function of state (rather than a process quantity).

P-V diagram work question

$$Work \equiv \int P dV$$
$$\Downarrow$$

area under the curve
for each process.



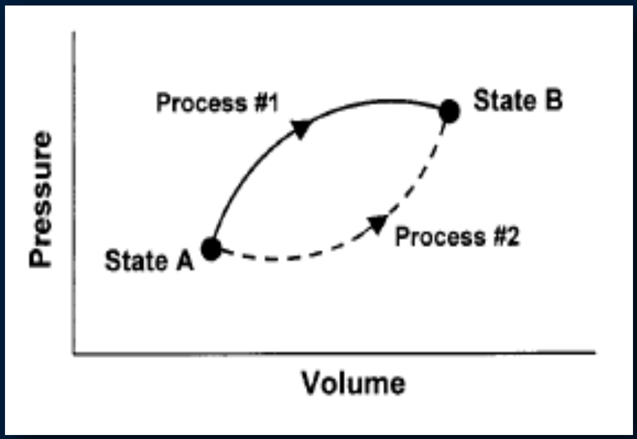
I n t e g r a l

Work ≡ ∫ PdV → Area under P-V curve → Depends on path

While characterized as a physics difficulty, could the underlying math also be a factor?

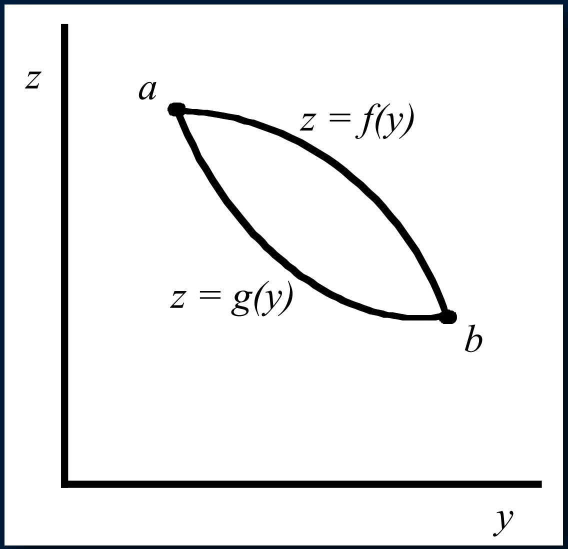
“Physics-less physics question”

E. Pollock, B. Bucy, JRT, D. Mountcastle



Compare works done

Work ≡ ∫ PdV

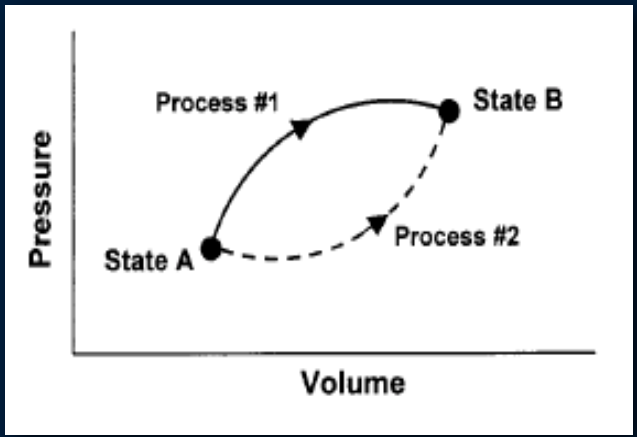


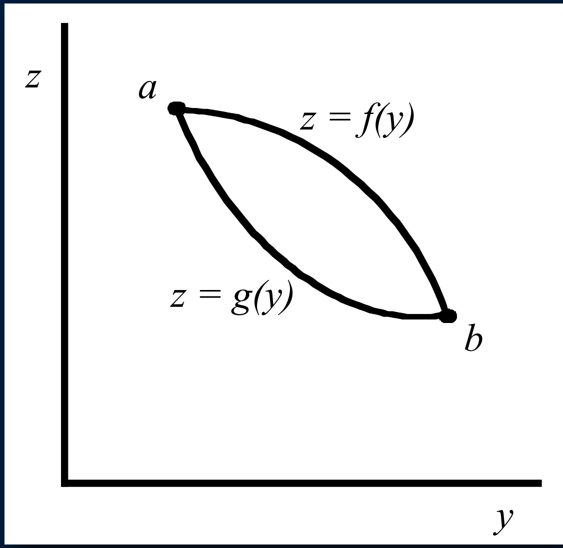
Compare integrals

Area under curve

"Physics-less physics question"

E. Pollock, B. Bucy, JRT, D. Mountcastle





Compare works done

Work ≡ ∫ PdV

Compare integrals

Area under curve

Some students (~20%) stated **integrals were different**, but **works were equal**:
math: area under curve
physics: state-function / path-independence;
"assuming zero dissipative processes";
"assuming conservative force."

"Physics-less physics question"

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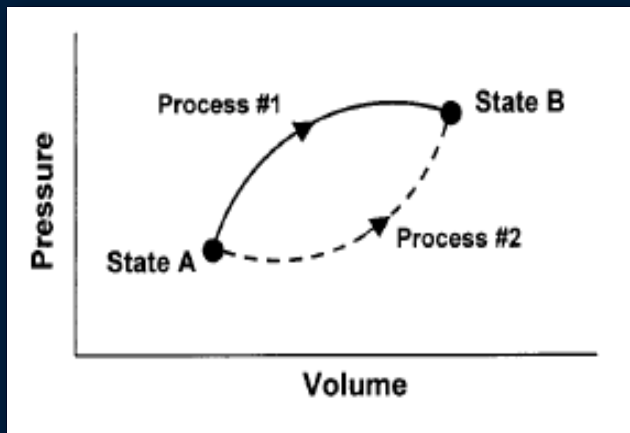
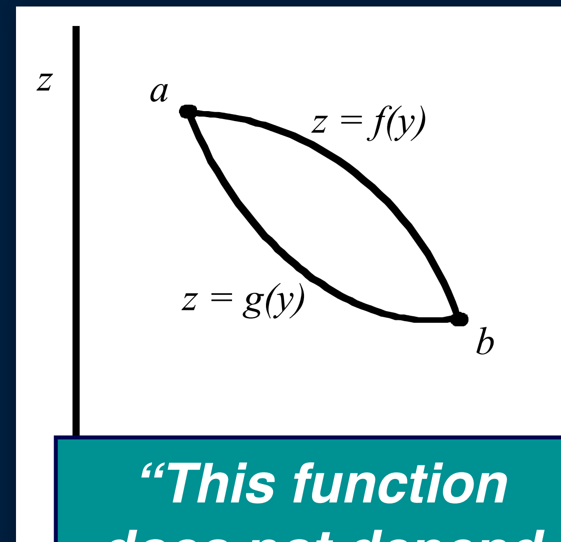


Compare works done

Work ≡ ∫ PdV



Cor

"This function does not depend on the path, but only on the endpoints."

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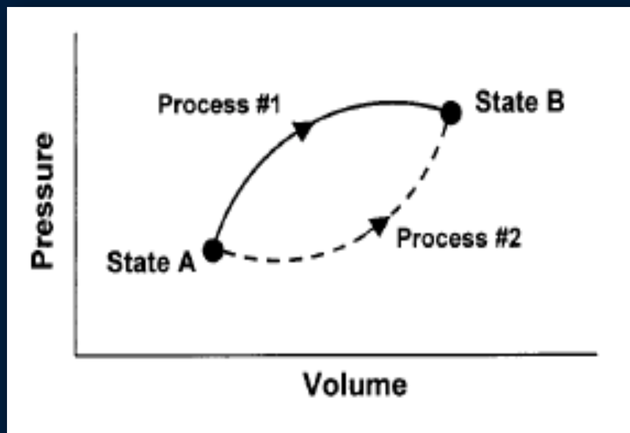
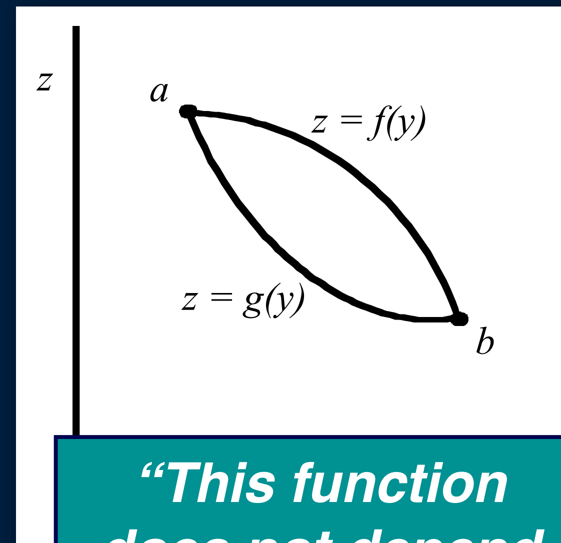


Compare works done

Work ≡ ∫ PdV



Cor

"This function does not depend on the path, but only on the endpoints."

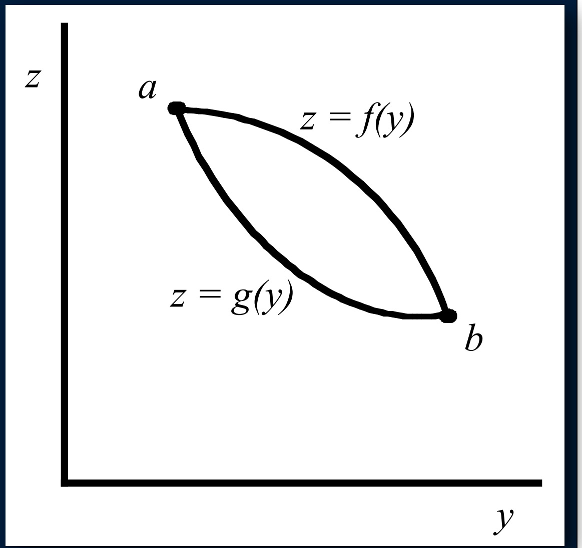
Many of the students who stated that the works *were equal also stated that the* integrals *were equal:*
Suggests some mathematical difficulties

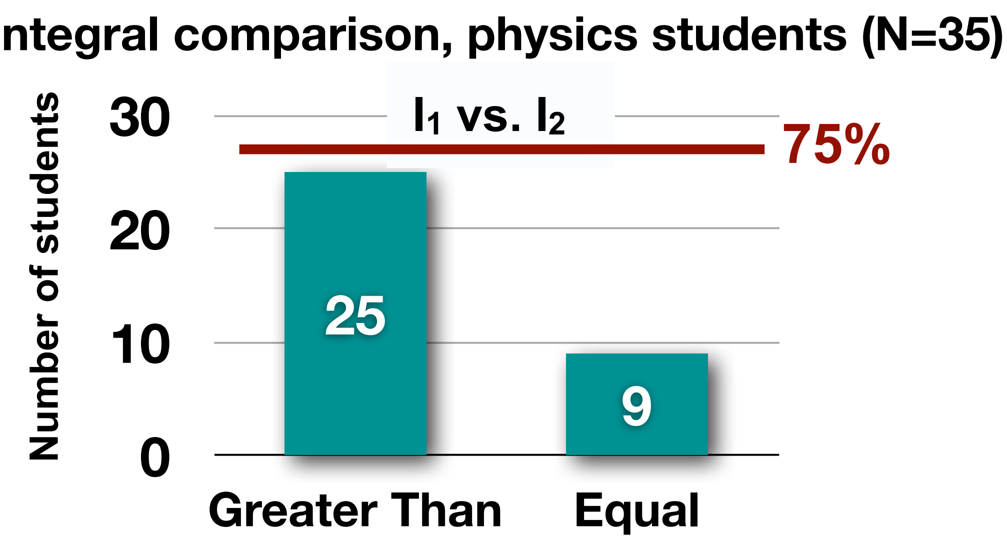
Context for research: extended

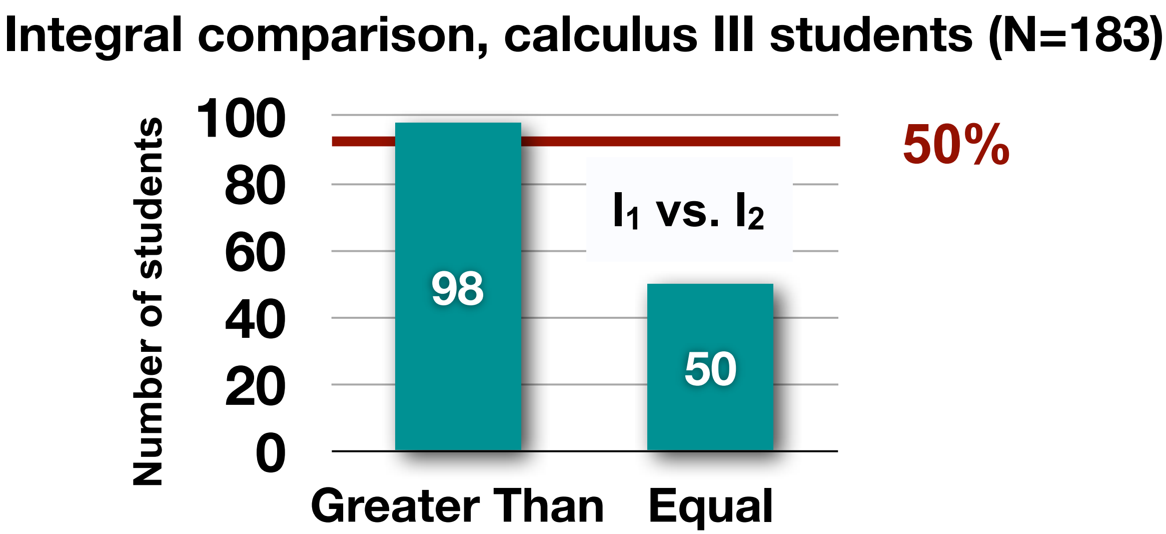
- Advanced undergraduate thermal physics courses;
  3 hours/week for semester
  - *Physical Thermodynamics*
  - *Statistical Mechanics*
  - Mathematics preparation

    All students had completed at least 4 mathematics courses, including *Calculus III*
- *Calculus III* : Multivariable calculus

Comparison of results: representational features and population

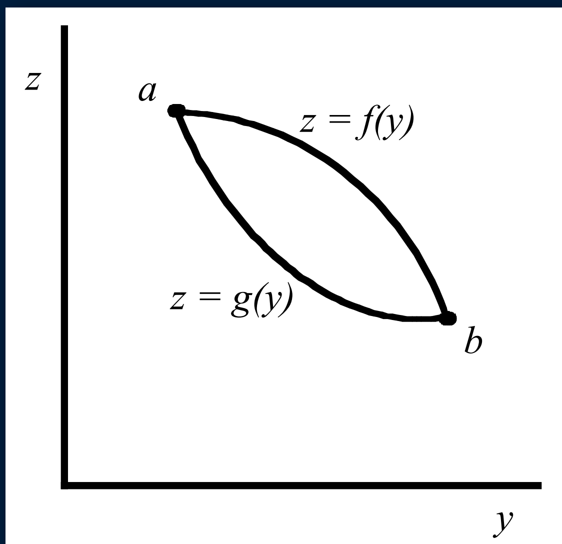






Effect of representation on student interpretation

In individual student interviews (E. Pollock)



integrals equal *due to symmetry*

↓

length of the path
and not the area under the curve

↓

"Line integral" interpretation

Colleagues in math education suggested this response/reasoning when presented

Why interdisciplinary communication matters

Context for research: extended

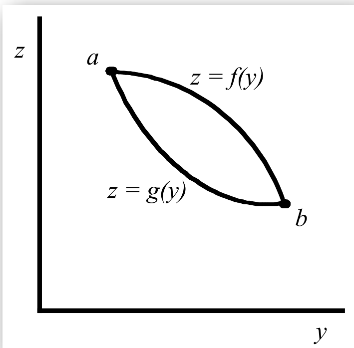
- Advanced undergraduate thermal physics courses;
  3 hours/week for semester
  - *Physical Thermodynamics*
  - *Statistical Mechanics*
  - Mathematics preparation

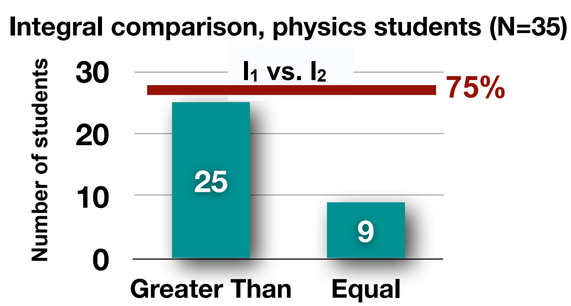
    All students had completed at least 4 mathematics courses, including *Calculus III*
- *Calculus III* : Multivariable calculus
- *Calculus II* : Integral calculus
- Introductory calc-based physics 2: Elec & Mag, Optics

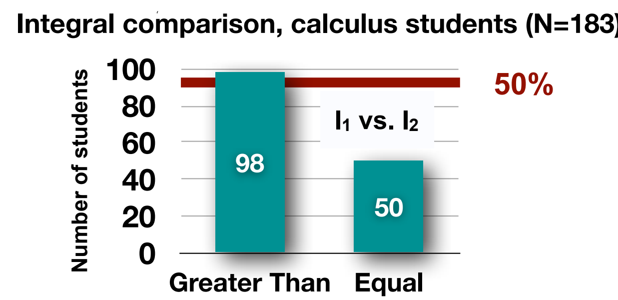
  Calculus II is co-requisite

Comparison of results: representational features and population

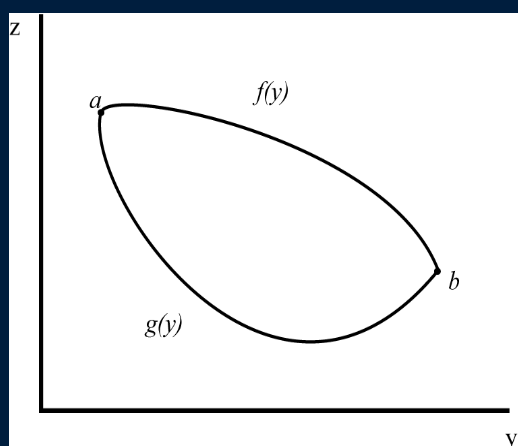
$$I_1 \equiv \int_a^b f(x)\,dx \qquad I_2 \equiv \int_a^b g(x)\,dx$$







Pollock, Thompson, Mountcastle *2007 PERC;* Christensen & Thompson, *Proc. 13th RUME* (2010)

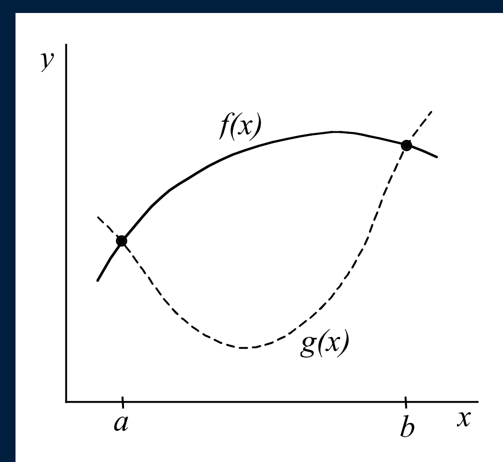


T. Wemyss

Comparison

Calc 2: 28 / 31 (90%)

Calc 3: 33 / 56 (58%)



R. Bajracharya, T. Wemyss

Physics and Calculus 3 **~70% for both**

Wemyss, Bajracharya, Thompson, Wagner, *Proc 14th RUME (2011)*

The transfer problem

Traditional question of transfer

How is knowledge learned in one situation able to be applied to a later, contextually different situation?

Common answer:

Transfer is supported by the acquisition or construction of *abstract*, *decontextualized* knowledge.

Abstract knowledge allows one to strip away irrelevant contextual or "surface features" of a problem or situation in order to see its underlying abstract or deep "structure."

(See, for example, Chi, Feltovich, & Glaser, 1981; Forbus, Gentner, & Law, 1995; Gick & Holyoak, 1980, 1983; Judd, 1908; Reed, 1987, 1993; Singley & Anderson, 1989; Wertheimer, 1945)

The transfer problem

Traditional question of transfer

How is knowledge learned in one situation able to be applied to a later, contextually different situation?

My question of interest

How does an individual come to identify two different problems or situations as (mathematically) alike?

The transfer problem

Some things I think I've learned:

1. The traditional "abstract knowledge" answer doesn't work.
2. It's a lot more complicated than that.
3. Context matters, and considerations of context are essential to both teaching and learning.
4. Context is in the eye of the beholder: What is an irrelevant "surface feature" to one may significantly engage the conceptual understanding of another.

A different approach

Transfer in pieces

(Wagner 2006, 2010)

1. Based on diSessa's (1993) "knowledge in pieces" epistemology.
2. A complex knowledge systems approach.
   - Some types of knowledge are made up of complex systems of many different types of knowledge elements, often sensitive to context.
   - Contextual features can cue particular collections of knowledge while failing to cue others.
   - Seeing the same concept or principle in two different situations may require the use of two different collections of knowledge elements, rather than a single "abstract" knowledge structure!

A different approach

Transfer in pieces

(Wagner 2006, 2010)

Some implications:

1. Learning to see the "same thing" across multiple contexts requires the construction and (re)organization of complex systems of knowledge, so it takes time.
2. The contexts in which learning takes place matter. From a TiP perspective, we should not expect even good performance in Calculus to transfer “automatically” into Physics applications.
3. Contextual roadblocks to learning can be very idiosyncratic.

Physics & Calculus

Physics & Calculus

Physicsless-physics

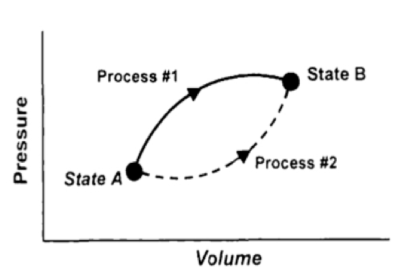
John Thompson and colleagues have been examining the relationship between students' understanding of physics and calculus as undergraduates study Thermodynamics.

Some of their data consider students' responses to explicitly stated physics problems along with responses to "physicsless-physics questions," or "physics questions that are completely stripped of their context" (Christensen & Thompson, 2010).

Physics & Calculus

Physics context

The Pressure-Volume (P-V) diagram represents a system consisting of a fixed amount of ideal gas that undergoes two ***different*** processes in going from state A to state B:

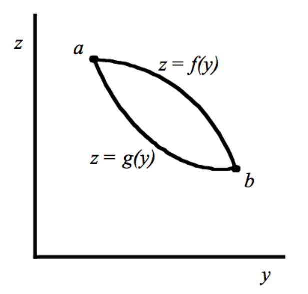


Is the work done by the system for Process #1 ***greater than***, ***less than***, or ***equal to*** that for Process #2?

Physics & Calculus

Physics-less context

Two paths have been traced out on the z-y graph shown below and are labeled Path 1 and Path 2. Both paths start at point a and end at point b. Consider the integrals $I_1 = \int_{a\,\text{Path1}}^{b} z\,dy$ and $I_2 = \int_{a\,\text{Path2}}^{b} z\,dy$, where I_1 is taken over Path 1 and I_2 is taken over Path 2.

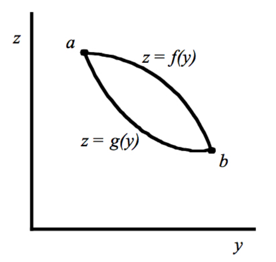


Is the absolute value of the integral I_1 ***greater than***, ***less than***, or ***equal to*** the absolute value of the integral I_2, or is there ***not enough information to decide***?

Physics & Calculus

What "surface features" might interfere with students' reasoning in the "physics-less" problem?

- The degree to which the physics-less questions are, indeed, physics-less is arguable.



Physics & Calculus

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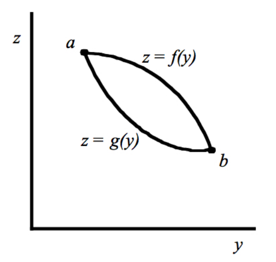


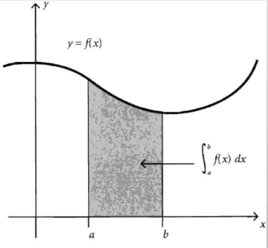
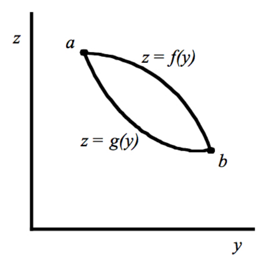


Figure 17.6

Standard notation within Mathematics

Physics & Calculus



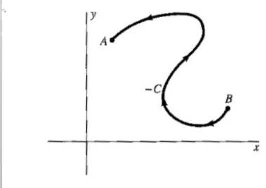
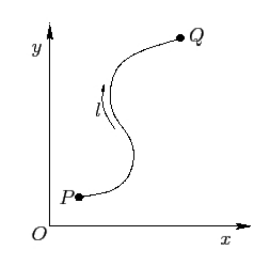


FIGURE 14.2.6

$$\int_{-C} f\,ds = \int_C f\,ds$$

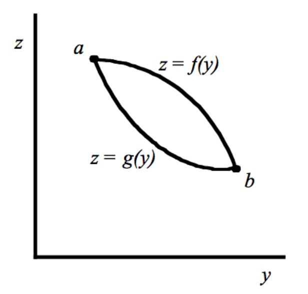


Typical mathematical representations for line/path integrals.

Physics & Calculus

Physics-less context

Two paths have been traced out on the z-y graph shown below and are labeled Path 1 and Path 2. Both paths start at point a and end at point b. Consider the integrals $I_1 = \int_{a_{\text{Path 1}}}^{b} z\,dy$ and $I_2 = \int_{a_{\text{Path 2}}}^{b} z\,dy$, where I_1 is taken over Path 1 and I_2 is taken over Path 2.



Is the absolute value of the integral I_1 ***greater than***, ***less than***, or ***equal to*** the absolute value of the integral I_2, or is there ***not enough information to decide***?

Of research interest...

- Context matters.
  - What are often considered irrelevant "surface features" by experts can be the source of genuine conceptual stumbling blocks to students.
  - Different surface features can also cue a variety of useful knowledge resources that become associated and integrated in knowledge networks more likely to function across contexts.
- Learning for transfer is not a matter of acquiring abstract understandings that *overlook* contextual differences, but of constructing networks of knowledge resources that *accommodate* them.
- "Transfer in pieces" offers a theoretical perspective that both predicts and explains many of the types of behavior seen in these examples, as well as why transfer can be a lengthy, complex process.

Of research interest. . .

- Face it: If knowledge of abstract structures and principles were the source of transfer, traditional lecture classrooms would be wildly successful.
- Abstract principles serve scientific communities well in constructing broad theories, but that does not mean that they reflect the mechanisms of human cognition.
- Exposure to the use of concepts/principles across multiple concepts is important, but *not* just for practice. The types of contexts in which students make use of different ideas can evoke rich networks of knowledge resources.

TRUSE Mini-grant: Activities and outcomes

- Fall 2010
  - JFW gives talk at UMaine, initial perusal of data
- Spring 2011
  - RRB administers more written questions, interviews
  - JRT, TMW, RRB, JFW give talk at RUME 2011
  - TMW, RRB, JRT, JFW publish RUME proceedings: "Student understanding of integration in the context and notation of thermodynamics: Concepts, representations, and transfer"

TRUSE Mini-grant: Activities and outcomes

- Summer 2011
  - JRT organizes invited session at AAPT Summer Meeting;
    JFW speaks, includes Maine data.
  - JFW, JRT, C. A. Manogue co-facilitate invited “Poster Gallery“ at PERC 2011:

    Representation Issues:
    Using Mathematics in Upper-Division Physics

    JFW serves as discussant in session.
  - JFW, CAM, JRT write invited article in 2011 PERC Proceedings: “Representation issues: Using mathematics in upper-division physics“

TRUSE Mini-grant: Activities and outcomes

- Fall 2011:
  - JRT gives talk at Xavier; examine recently collected data
  - full realization that existing data do not fit our needs
  - discussion of next steps

Data needs

- Careful transfer analysis requires detailed data of the same students working with “the same concept” across different contexts.
- Data available from the work of JRT and colleagues, though often nicely detailed, do not suffice for cross-contextual analysis. For example, they look at students’ understanding of the definite integral in different *mathematical* contexts, but we have not gathered comparable interview data of the same students within the context of *physics* problems.
- Transfer-in-pieces analysis focuses on students caught in the act of learning–making cross-contextual connections for the first time. Thompson’s data focus more on students’ understanding at a point in time without instruction or efforts to engage students in cross-contextual reasoning.

Emerging perspectives

- Maine data suggest a need to consider more carefully the interrelated aspects of learning physics and learning mathematics.

- Students' confusion of "area" integrals with "line" integrals are traditionally interpreted as distractions due to "surface features" of the problems. We think it likely that these surface features mask deeper conceptual issues:

  - Difficulties understanding the mathematics: Students often leave Calculus with a static (area model) understanding of the definite integral rather than an accumulation model.

  - Difficulties understanding the physics: Use of the word *path* in a physics context takes on new meaning, unlikely to have been seen or used in Calculus where "paths" most often refer to trajectories in space.

  - Simultaneous difficulties: Jumping to a line integral where an area integral is needed points to lack of conceptual engagement with the "meeting place" of both the mathematics and the physics.

Emerging perspectives

- It is likely too limiting to ask how the math is or isn't "transferred" to the physics. Rather, we need to consider how new understandings of both the mathematics and the physics co-emerge in the context of learning physics.

- What do students learn about mathematics as they learn physics?

TRUSE Mini-grant: Next steps

Assemble grant proposal(s); may be part of larger proposal with additional collaborators

- Integrals as context for evidence of transfer in pieces from math to physics
- Set up interview protocol to interview *same students* about analogous scenarios in both mathematics and physics contexts
- Potential finding sources:
  - NSF REESE (July)
  - Spencer Foundation (“small“ [July/Fall] and/or “large“ [preliminary proposal: October])